Homework

Basics in Mathematics – Geometry

To be handed in by Thursday September 30th, 2021, at 10:30am

Exercise 1. Show that the collection

$$\tau = \{U \subset \mathbb{R} \mid \mathbb{R} \smallsetminus U \text{ is either finite or } \mathbb{R}\}$$

is a topology on $\mathbb{R}$.

Exercise 2. Let (X, τ) be a topological space and Y a subset, with the subspace topology. Show that the inclusion map

$$\begin{aligned} \iota : Y &\hookrightarrow X \\ y &\mapsto y \end{aligned}$$

is continuous.

Exercise 3. Let (X, τ) and Y, σ be the topological spaces given by $X = \{a, b, c\}$, $\tau = \mathcal{P}(X)$, $Y = \{x, y, z\}$, $\sigma = \{\emptyset, \{x\}, Y\}$. Consider the map

$$\begin{aligned} f : X &\to Y \\ a &\mapsto x \\ b &\mapsto y \\ c &\mapsto z. \end{aligned}$$

Show that f is continuous. Is f a homeomorphism? If yes, prove it. If not, justify why.

Exercise 4. Consider the following relations. Are they equivalence relations? If yes, prove it. If not, find an example showing that one of the properties of equivalence relations fails.

1. On $X = \{\text{squares in } \mathbb{R}^2\}$, the relation *has the same area as.*
2. On $X = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ function}\}$, the relation $\sim$ given by $f \sim g$ if for every $x \in \mathbb{R}$, $f(x) \leq f(y)$.